RADIO-FREQUENCY CONDUCTIVITY OF IONIZED GASES IN MAGNETIC FIELD

R. N. GUPTA AND S. K. MANDAL

DEPARTMENT OF PHYSICS, NORTH BENGAL UNIVERSITY,

RAJA RAMMOHUNPUR, DARJEELING.

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ABSTRACT. The radio-frequency conductivity of ionized gases (air and Carbondioxide) has been measured within a pressure range of a few microns of Hg to .3 mm of Hg in presence of a magnetic field varying from 0 to 700 Gauss, and a frequency of 10.6 Mc/sec, the discharge being excited by a transformer. It has been observed that conductivity decreases in presence of magnetic field for all values of pressure and the pressure at which the conductivity becomes a maximum increases with the increase of magnetic field. The results can be explained fairly well by an extension of the theory put forward by Gilardini (1959) and the quantitative agreement is also satisfactory. The introduction of the effect of equivalent pressure generally gives results in wide divergence with experimental results and hence it is concluded that for values of (H/P) employed in this case, the equivalent pressure concept does not hold. The reasons for the failure of equivalent pressure expression in this case have been discussed.

INTRODUCTION

In a previous paper (Sen and Ghosh, 1966) a method has been described to measure the radio-frequency conductivity of ionised gases and a study has been made regarding the interaction of radio-frequency waves with ionised gases. Since the presence of a magnetic field changes the various characteristics of a discharge, it is natural to suppose that the radio-frequency conductivity of an ionized gas will also change in presence of a magnetic field. Conductivity of ionized gases such as air, nitrogen and hydrogen in a magnetic field was measured by Ionescu and Mihul (1932) for pressure greater than 10^{-3} mm of Hg who found that maxima other than those due to free electrons could be obtained. With very intense fields, only the vibration due to free electrons remained, the others disappearing and the values of the magnetic field giving maximum conductivity varied with pressure. A theory regarding the variation of radio-frequency conductivity with magnetic field was proposed by Appleton and Boohariwala (1935) who showed that the real part of radio-frequency conductivity in a magnetic field is given by

$$\sigma_{rH} = \frac{ne^2}{m} \cdot \frac{\nu_c(\omega^2+\omega_b^2+\nu_c^2)}{(\omega^2+\omega_b^2+\nu_c^2)^2-4\omega^2\omega_b^2} . \quad \ldots \quad (1)$$

n is the number of electrons per unit volume and ν_c the collision frequency, ω is the angular frequency of the applied field and $\omega_b = \dfrac{eH}{m}$; from graphical analysis, it was shown by the authors that the value of ν_e for which the conductivity becomes a maximum is obtained when $\nu_c = 0$ which is anomalous and further the experimental results obtained by the authors were not supported by the theory developed; but it was conclusively shown that the magnetic field has a marked influence on the pressure at which the conductivity becomes a maximum and the value of the conductivity changes when the magnetic field is applied. A general theory regarding the variation of radio-frequency conductivity of ionized gases and its variation with pressure and the magnetic field has been worked out by Gilardini (1959) who derived the expression for the conductivity of an ionized gas under the following assumptions:

(a) when the distribution function is predominently spherically symmetrical in velocity space but not necessary Maxwellian.

(b) when the electron collison frequency is an arbitrary function of electron velocity. The value of the complex conductivity is given by

$$\sigma = \frac{e^2 n}{m} \cdot \frac{1}{\nu_e + j\omega}$$

In presence of magnetic field he has defined two conductivities; a conductivity σ_c for the right-handed polarization and a conductivity σ_0 for the left-handed polarization where

$$\sigma_e = \frac{e^2 n}{m} \left[\frac{1}{\nu_c + j(\omega - \omega_b)} \right].$$

and

$$\sigma_0 = \frac{e^2 n}{m} \left[\frac{1}{\nu_c + j(\omega + \omega_b)} \right.$$

and the conductivity in the direction of the field is given by

$$\sigma_H = \frac{1}{2}(\sigma_c + \sigma_0).$$

and

$$\sigma_H = \frac{e^2 n}{m} \left[\left\{ \frac{\nu_c}{\nu_e^2 + (\omega - \omega_b)^2} + \frac{\nu_c}{\nu_c^2 + (\omega + \omega_b)^2} \right\} \right.$$

$$\left. - j \left\{ \frac{(\omega - \omega_b)}{\nu_c^2 + (\omega - \omega_b)^2} + \frac{(\omega + \omega_b)}{\nu_c^2 + (\omega + \omega_b)^2} \right\} \right]$$

so that real part of the conductivity is given by

$$\sigma_{rH} = \frac{e^2 n}{m} \left[\frac{\nu_c}{\nu_c^2 + (\omega - \omega_b)^2} + \frac{\nu_c}{\nu_e^2 + (\omega + \omega_0)^2} \right]$$

and after simplification it reduces to the result obtained earlier by Appleton and Boohariwalla (1935)

$$\sigma_{rH} = \frac{e^2 n}{m} \frac{\nu_c[\nu_c^2+\omega_b^2+\omega^2]}{(\nu_c^2+\omega^2+\omega_b^2)^2-4\omega^2\omega_b^2}$$

Though some measurements of radio-frequency conductivity have been carried our earlier it is felt necessary that a thorough and systematic experimental measurement of radio frequency conductivity of ionized gases in a magnetic field will yield some data which can be utilized for the verification of the theory advanced by Gilardini (1959) or by Appleton and Boohariwall (1935). Also it will be of interest to study the variation of radio-frequency conductivity in a magnetic field and to see how the pressure at which the conductivity becomes a maximum varies with the application of the magnetic field. An idea regarding the interaction of the magnetic field with the ionized gases can thus be obtained. With this object in view the present work has been undertaken and the paper reports the results obtained experimentally in case of air and carbondioxide in presence of magnetic field varying from 0 to 700 gauss and the pressure varying from a few microns to 300 microns.

EXPERIMENTAL PROCEDURE

The radio-frequency conductivity of ionized gases such as air and carbondioxide has been determined in the same way as has been done by (Sen and Ghosh, 1966). Pure and dry air has been used and carbondioxide has been prepared by letting a saturated solution of oxalic acid (analytical grade) in water fall drop by drop in to a saturated solution of sodiumbicarbonate (analytical grade). The evolved carbondioxide was passed through phosphorus pentoxide to remove water vapour. The magnetic field has been supplied by an electromagnet and the lines of force are perpendicular both to the direction of the field and to the length of the discharge tube. Keeping the magnetic field constant at a particular value, the pressure of the gas has been varied and the conductivity of the gas determined for various values of the pressure, and the same procedure has been repeated for various values of the magnetic field. The experiment has been repeated a large number of times and the results have been found to be consistent. The pressure of the gas has been measured as was done in the previous papers, (Sen *et. al.*, 1962, a, b). The frequency of the applied field as measured by a wide band communication receiver was 10.6 Mc/sec and measurements were taken for the value of the discharge current of 20mA. The values of the magnetic field have been measured accurately by a calibrated Fluxmeter.

RESULTS AND DISCUSSION

The variation of radio-frequency conductivity against pressure has been plotted in case of air and carbondioxide for different values of the magnetic field in Fig. 1 and Fig. 2; also the conductivity pressure curve without magnetic field

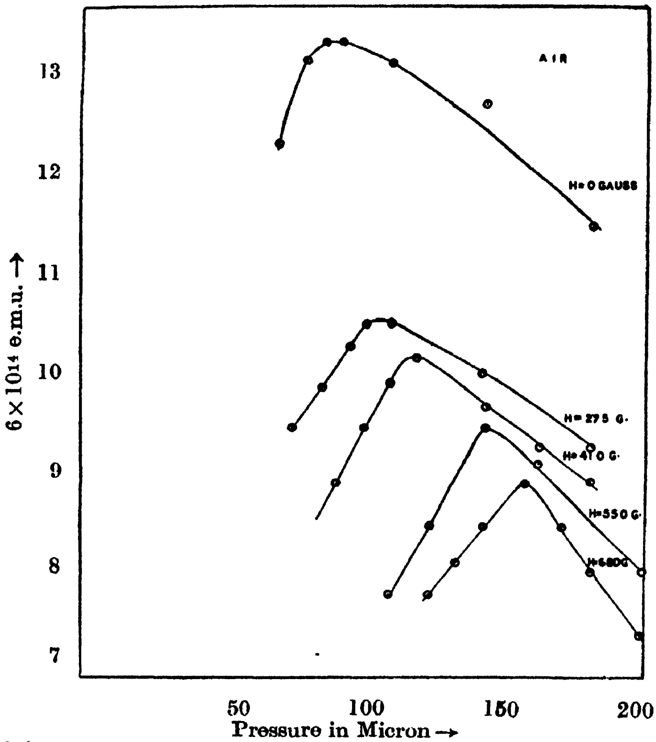

Fig. 1. Conductivity of ionised air against pressure for different values of the magnetic field.

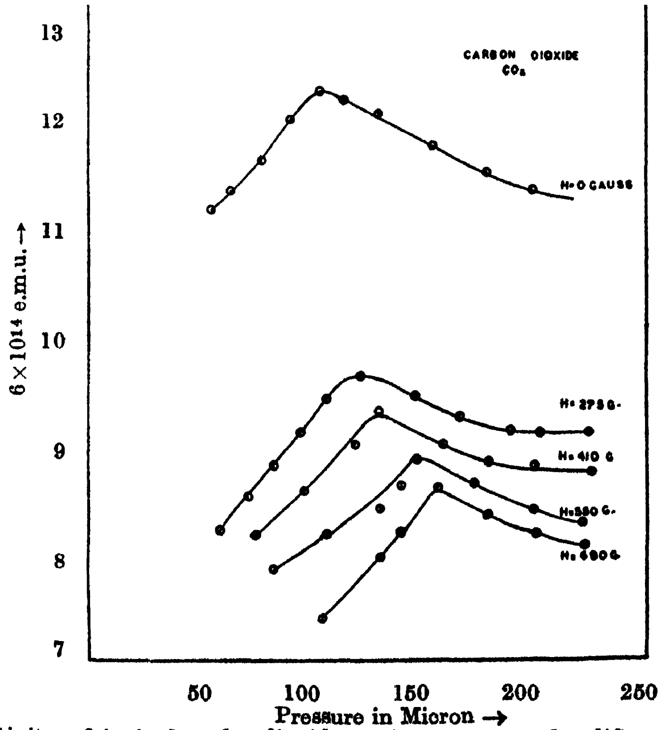

Fig. 2. Conductivity of ionised carbondioxide against pressure for different values of the magnetic deld.

has been given for comparison. It is observed that the value of σ_r is smaller when magnetic field is present than that without field for all values of pressure and the pressure at which the conductivity becomes a maximum always shifts to higher pressure when the magnetic field is increased. That the real part of r.f. conductivity will be smaller in presence of magnetic field than when the field is absent is evident from the following considerations; we have

$$\sigma_r = \frac{ne^2}{m} \cdot \frac{\nu_c}{\nu_c^2+\omega^2}$$

and

$$\sigma_{rH} = \frac{ne^2}{m} \cdot \frac{\nu_c(\nu_c^2+\omega^2+\omega_b^2)}{(\nu_c^2+\omega^2+\omega_b^2)^2-\omega^2\omega_b^2}\,.$$

so that when the magnetic field employed is of the order of 200 gauss, we get

$$\omega = 3.52\times 10^9 \text{ radians}$$

and for $H = 300$ Gauss, $\omega = 5.28\times 10^9$ radians

whereas the frequency of the applied field is of the order of 2.95×10^6 cycles/sec or 1.852×10^7 radians; we can therefore neglect ω in comparison to ω_b and hence obtain

$$\sigma_{rH} = \frac{ne^2}{m} \cdot \frac{\nu_c(\nu_c^2+\omega_b^2)}{(\nu_c^2+\omega_b^2)^2-4\omega^2\omega_b^2}\,.$$

then

$$\frac{\sigma}{\sigma_{rH}} = \frac{(\nu_c^2+\omega_b^2)^2-4\omega^2\omega_b^2}{(\nu_c^2+\omega_b^2)(\nu_c^2+\omega^2)}\,.$$

and neglecting $4\omega^2\omega_b^2$ in comparison to $(\omega_b^2+\nu_e^2)^2$ we get

$$\frac{\sigma}{\sigma\pi_H} = \left(\frac{1+\omega_b^2/\nu_c^2}{1+\omega^2/\nu_c^2}\right)$$

and since $\omega_b >> \omega$, $\sigma/\sigma_{\gamma H}$ will be greater than unity as is actually found to be the case in the range of pressure investigated. Considering from the physical point of view it is seen that due to the presence of magnetic field the effective mean free path is shortened and now there is a greater number of collisions which results in a net reduction of the number of drifting electrons contributing to the conductivity current and hence the conductivity decreases. However, in the discussion which follows it will be assumed that the number of electrons per unit volume is the same in the presence of magnetic field as in its absence, because it is clearly observed in the course of experiment as well as from theoretical considerations that there is a gradation of concentrations of electrons in the path of the radio-frequency field but the average number approximately remains the same.

The values of pressure at which the conductivity becomes maximum in case of air and carbondioxide have been obtained from the curves of Fig. 1 and 2 and entered in Table I.

TABLE I

Gas	Magnetic field in gauss	Value of maximum conductivity $\times 10^{14}$ e.m.u.	Corresponding pressure as from experiment in micron.	(P_H) min from equ. (2) (microns) (4)	(P_H) min from equn (4) in micron
Air	0	13.175	92		
	275	10.475	112	115.7	92.56
	410	10.125	123	118.8	95.04
	550	9.175	150	130.02	104.016
	680	8.923	170	136.66	109.33
Carbondioxide	0	12.6	108		
	275	9.85	125	140.7	112.56
	410	9.45	140	146.7	117.36
	550	9.00	155	154.0	123.2
	680	8.725	162	157.9	127.12

It is evident that the maximum value of conductivity diminishes and the pressure at which the conductivity becomes a maximum shifts to higher values whth the application of the magnetic field. To explain this it is observed that since $\omega_b >> \omega$ for the values of magnetic field employed,

$$\sigma_{rH} = \frac{ne^2}{m} \qquad \nu_c^2+\omega_b^2$$

and putting
$$\nu_c = \frac{v_r}{\lambda} = \frac{v_r P}{L}$$

where L is the mean of free path of the electron in the gas at a pressure of 1*mm* v_γ the velocity of the electrons we get,

$$\sigma_{rH} = \frac{ne^2}{m} \cdot \frac{L}{v_r} \qquad \overline{P^2+C_1H^2}$$

where
$$C_1 = \left(\frac{e}{m} \cdot \frac{L}{v_r}\right)^2$$

and σ_{rH} will be a maximum with respect to P

when $$(P_H)_{max}^2 = C_1 H^2.$$

where $(P_H)_{max}$ is the pressure at which the conductivity becomes a maximum in presence of magnetic field.

then $$(\sigma_{rH})_{max} = \frac{1}{2} \cdot \frac{ne^2}{m} \cdot \frac{L}{v_r} \cdot \frac{1}{(P_H)_{max}} \cdot$$

and $$(\sigma_r)_{max} = \frac{1}{2} \cdot \frac{ne^2}{m} \cdot \frac{L}{v_r} \cdot \frac{1}{P_{max}} \cdot$$

so that $$(P_H)_{max} = \frac{(\sigma_r)_{max}}{(\sigma_{rH})_{max}} \cdot P_{max} \cdot \qquad (2)$$

The values of $(\sigma_r)_{max}$ and $(\sigma_{rH})_{max}$ as well as P_{max} can be obtained from experimental data and hence $(P_H)_{max}$ can be calculated. The results calculated from equation (2) have been entered into the fifth column of the Table 1; the agreement is quite satisfactory considering the simplifications involved in the deduction of the equation. The result also shows that the pressure at which the radio-frequency conductivity becomes maximum always shifts to higher values as the magnetic field is increased. The above deduction cannot be taken as rigorous because in presence of a magnetic field the actual pressure is changed to an effective pressure as has been shown by Blevin and Haydon, (1958)

$$P_H = P\sqrt{1+C_1H^2/P^2}.$$

where $$C_1 = \left(\frac{e}{m} \cdot \frac{L}{v_r}\right)^2.$$

Due to this change of pressure the collision frequency also changes. Gilardini (1959) in developing his theory has not taken this change into consideration and assumed that the collision frequency is same both with the without magnetic field. Hence

$$\sigma_{rH} = \frac{ne^2}{m} \cdot \frac{L}{v_r} \cdot \frac{[P^2+C_1H^2]^{\frac{1}{2}}}{P^2+2C_1H^2} \qquad \ldots \quad (3).$$

he condition for obtaining the maximum value of σ_{rH} when its variation with pressure is taken into consideration is obtained from equation (3). This occurs when $P = 0$ which is anomalous. It may be recalled that Appleton and Booharibwalla (1935), without taking into consideration the concept of equivalent pressure also came to the same conclusion from graphical analysis. Consequently if we adopt the Blevin and Haydon expression in this case, it leads to result which is anomalous. Of course the validity of Belvin and Haydon's expression has previously been tested in case of air by Sen and Ghosh (1961) where it was shown that for values of pressure less than 150μ and magnetic field of the order of 100 gauss the expression gives values of equivalent pressure different from the actual pressure. Since the pressure at which maxima are occuring is in a region greater than 100μ, and the magnetic field employed is also large it can be conjectured that Blevin and Haydon's expression does not hold in the region of pressure where maxima are occuring.

Townsend and Gill (1937) on the otherhand deduced that

$$\mu_H = \frac{\mu}{1+\omega_b{}^2\tau^2}.$$

where μ_H is the mobility of electrons in the magnetic field H; τ is the time between successive collisions and $\omega_b = eH/m$. It can be deduced from the above relation that

$$P_H = P\left[1+C_1\frac{H^2}{P^2}\right]$$

and hence $$\sigma_{rH} = \frac{ne^2}{m}\cdot\frac{L}{v_r}\cdot\frac{P[P^2+C_1H^2]}{[(P^2+C_1H^2)^2+C_1H^2P^2]}.$$

and σ_{rH} is maximum when $(P_H)_{max} = C_1H^2$.

then $$(\sigma_{rH})_{max} = \frac{ne^2}{m}\cdot\frac{L}{v_r}\frac{1}{(P_H)_{max}}\cdot\frac{2}{5}.$$

and as $$(\sigma_r)_{max} = \frac{ne^2}{m}\cdot\frac{L}{v_r}\cdot\frac{1}{P_{max}}\cdot\frac{1}{2}.$$

we get $$(P_H)_{max} = \frac{(\sigma_r)_{max}}{(\sigma_{rH})_{max}}\cdot\frac{4}{5}\cdot(P_H)_{max}. \qquad \ldots \quad (4)$$

The results calculated from equation (4) are entered into the last column of Table I. It is observed that results obtained with equation (4) are in wide disagreement with the experimental results both in the case of air and carbondioxide. It is thus evident that the concept of equivalent pressure whether from Blevin and Haydon expression or from Townsend expression can not lead to any imporvement in the theoretical deduction.

It can thus be conluded that in case of air and carbondioxide the simple theory put forward by Gilardini can explain the results quite well specially when the gyro-frequency is far removed from the frequency of the measuring field but this treatment is over simplified. Further it has been shown that the inclusion of the concept of equivalent pressure does not lead to any better results. In fact Haydon (1961) has discussed the limitation of the equivalent pressure concept in which he found different values of C_1 for Hydrogen by plotting (α_H/α_0) where α is the first Townsend coefficient against values of (H/E) varying from 0 to 2.5 where E is the breakdown voltage. From this he has concluded that perhaps drift velocity is a linear function of (E/P) for small (E/P) values but varies as $(E/P)^n$ where $n > 1$ for large (E/P) values. The value of E/P in these experiments is of the order of 150 volts/cm mm of Hg and hence the linearity relation between the drift velocity and (E/P) on which the Blevin Haydon expression is based, may not hold good for the values of (E/P) used here. This may partly account for the failure of the conception of equivalent pressure in explaining the observed results. The limitations of equivalent pressure concept have also been discussed previously by Sen and Ghosh (1961) where it was shown that the expression is valid upto a pressure of 150μ when the magnetic field is of the order of 100 gauss. Since the magnetic field used here is much greater than 100 gauss, and the maxima are also occuring at pressures greater than 150μ, the Blevin Haydon expression cannot be expected to hold here. Experiments are in progress in this laboratory to measure the radio-frequency conductivity in other gases specially in inert gases and the results will be reported in future.

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